local area is constant regardless of signal bandwidth. We saw that a narrowband signal may undergo rapid fluctuations in a local area, whereas wideband signals typically have very little signal amplitude fluctuation over the same local area.

Three common types of wideband channel sounders were described, and measurements that are made to describe multipath channels were also presented. This chapter also presented important time and frequency relationships that provide metrics for determining the time dispersion (multipath delay) and frequency dispersion (variation due to motion) and how they relate to flat versus frequency selective fading channels. These metrics were shown to be valuable in the design of air interface standards for mobile radio environments.

The classical Clarke and Gans theory for Rayleigh fading propagation in a local area was given, and the resulting theory was used to determine the classical U-shaped spectrum for a mobile radio signal, as well as level crossing rates and average fade duration statistics.

Finally, a number of common statistical modeling techniques, including measurement-based tapped delay line models such as SIRCIM and SMRCIM, were presented. Statistical models based on measurements have the advantage of replaying impulse response samples that have similar statistics to actual field data, thereby providing accurate and realistic channel samples that can be used reliably to develop products and explore complex signal processing algorithms in the lab, without the need of actual working hardware.

This chapter concluded by presenting a new and powerful theory that uses knowledge of the distribution of multipath energy arriving azimuthally at a receiver antenna. Multipath shape factor theory provides an easy, intuitive, and accurate method for analyzing small-scale fading channels with non-omnidirectional multipath propagation. The theory also has many implications for the measurement of wireless channels. For example, fading along specific but perpendicular directions may be measured in a local area with a simple, non-coherent receiver to calculate various multipath angle-of-arrival and fading rate characteristics. Conversely, angle-of-arrival characteristics may be measured with a directional antenna to calculate local area fading behavior.

All of the above small-scale channel modeling topics are critical in the design of a practical air interface, as they impact the selection of modulation data rates and modulation methods for time varying mobile channels. Modulation techniques are the subject of the next chapter.

5.10 Problems

5.1 Determine the maximum and minimum spectral frequencies received from a stationary GSM transmitter that has a center frequency of exactly 1950.000000 MHz, assuming that the receiver is traveling at speeds of: (a) 1 km/hr; (b) 5 km/hr; (c) 100 km/hr; and (d) 1000 km/hr.

5.2 Describe all the physical circumstances that relate to a stationary transmitter and a moving receiver such that the Doppler shift at the receiver is equal to: (a) 0 Hz; (b) $f_{d_{max}}$; (c) $-f_{d_{max}}$; and (d) $f_{d_{max}}/2$.

5.3 Using first principles from linear system theory and the definition of the complex envelope, prove that Figure 5.3 is correct. That is, prove that for bandpass signals x(t) and y(t), Equation (5.9) is indeed valid. This is the key principle used in simulation and DSP.

5.4 Draw a block diagram of a binary spread spectrum sliding correlator multipath measurement system. Explain in words how it is used to measure power delay profiles.

(a) If the transmitter chip period is 10 ns, the PN sequence has length 1023, and a 6 GHz carrier is used at the transmitter, find the time between maximal correlation and the slide factor if the receiver uses a PN sequence clock that is 30 kHz slower than the transmitter.

(b) If an oscilloscope is used to display one complete cycle of the PN sequence (that is, if two successive maximal correlation peaks are to be displayed on the oscilloscope), and if 10 divisions are provided on the oscilloscope time axis, what is the most appropriate sweep setting (in seconds/division) to be used?

(c) What is the required IF passband bandwidth for this system? How is this much better than a direct pulse system with similar time resolution?

5.5 Given that the coherence bandwidth is approximated by Equation (5.39), show that a flat fading channel occurs when $T_s \geq 10\,\sigma_\tau$. Hint: Note that B_c is an RF bandwidth, and assume that T_s is the reciprocal of the baseband signal bandwidth.

5.6 If a particular modulation provides suitable BER performance whenever $\sigma_\tau / T_s \leq 0.1$ determine the smallest symbol period T_s (and thus the greatest symbol rate) that may be sent through RF channels shown in Figure P5.6, without using an equalizer.

5.7 If a baseband binary message with a bit rate R_b = 100 kbps is modulated by an RF carrier using BPSK, answer the following:

(a) Find the range of values required for the rms delay spread of the channel such that the received signal is a flat-fading signal.

(b) If the modulation carrier frequency is 5.8 GHz, what is the coherence time of the channel, assuming a vehicle speed of 30 miles per hour?

(c) For your answer in (b), is the channel "fast" or "slow" fading?

(d) Given your answer in (b), how many bits are sent while the channel appears "static"?

(e) A CDMA Rake receiver is able to exploit multipath when the channel is (circle all that apply)
(a) flat; (b) slow; (c) fast; (d) frequency selective

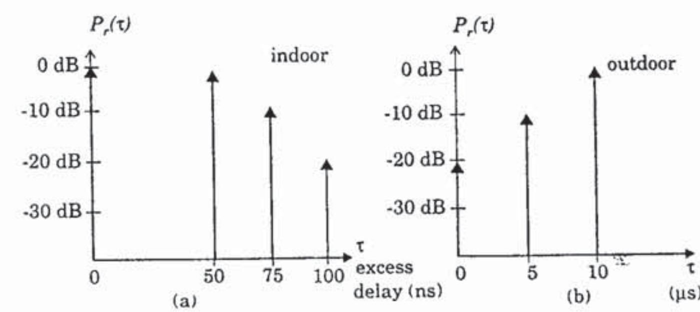

Figure P5.6 Two channel responses for Problem 5.6.

5.8 For the power delay profiles in Figure P5.6, estimate the 90% correlation and 50% correlation coherence bandwidths.

5.9 Approximately how large can the rms delay spread be in order for a binary modulated signal with a bit rate of 25 kbps to operate without an equalizer? What about an 8-PSK system with a bit rate of 75 kbps?

5.10 Given that a Rayleigh-faded mobile radio signal has a level crossing rate of $N_r = \sqrt{2\pi} f_m \rho e^{-\rho^2}$, find the value of ρ for which N_r is a maximum.

5.11 Given that the probability density function of a Rayleigh distributed envelope is given by $p(r) = \frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right)$, where σ^2 is the variance, show that the cumulative distribution function is given as $p(r < R) = 1 - \exp\left(\frac{-R^2}{2\sigma^2}\right)$. Find the percentage of time that a signal is 10 dB or more below the rms value for a Rayleigh fading signal.

5.12 The fading characteristics of a CW carrier in an urban area are to be measured. The following assumptions are made:

(1) The mobile receiver uses a simple vertical monopole.
(2) Large-scale fading due to path loss is ignored.
(3) The mobile has no line-of-sight path to the base station.
(4) The pdf of the received signal follows a Rayleigh distribution.

(a) Derive the ratio of the desired signal level to the rms signal level that maximizes the level crossing rate. Express your answer in dB.

(b) Assuming the maximum velocity of the mobile is 50 km/hr, and the carrier frequency is 900 MHz, determine the maximum number of times the signal envelope will fade below the level found in (a) during a 1 minute test.

(c) How long, on average, will each fade in (b) last?

5.13 A vehicle receives a 900 MHz transmission while traveling at a constant velocity for 10 s. The average fade duration for a signal level 10 dB below the rms level is 1 ms. How far does the vehicle travel during the 10 s interval? How many fades does the signal undergo at the rms threshold level during a 10 s interval? Assume that the local mean remains constant during travel.

5.14 An automobile moves with velocity $v(t)$ shown in Figure P5.14. The received mobile signal experiences multipath Rayleigh fading on a 900 MHz CW carrier. What is the average crossing rate and fade duration over the 100 s interval? Assume $\rho = 0.1$ and ignore large-scale fading effects.

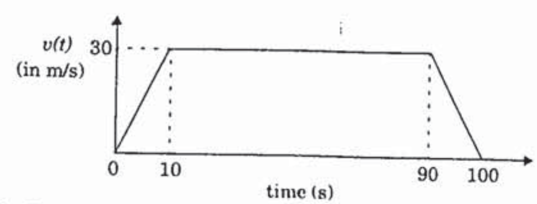

Figure P5.14 Graph of velocity of mobile.

5.15 For a mobile receiver operating at frequency of 860 MHz and moving at 100 km/hr

(a) sketch the Doppler spectrum if a CW signal is transmitted and indicate the maximum and minimum frequencies

(b) calculate the level crossing rate and average fade duration if $\rho = -20$ dB.

5.16 For the following digital wireless systems, estimate the maximum rms delay spread for which no equalizer is required at the receiver (neglect channel coding, antenna diversity, or use of extremely low power levels).

System	RF Data Rate	Modulation
USDC	48.6 kbps	π/4DQPSK
GSM	270.833 kbps	GMSK
DECT	1152 kbps	GMSK

5.17 In the Clarke and Gans model for small-scale fading, the E-field of the vertically polarized signal is given by

$$E_z(t) = E_0 \sum_{n=1}^{\infty} C_n \cos(2\pi f_c t + \theta_n)$$

where $\tau_n = 2\pi f_n t + \phi_n$ and

$$\sum_{n=1}^{N} \overline{C_n^2} = 1$$

and f_n is the Doppler shift of the nth plane wave, given by $f_n = v/\lambda \cos \alpha_n$.

(a) Write E_z in terms of narrowband in-phase and quadrature components $T_c(t)$ and $T_s(t)$ such that $E_z(t) = T_c(t)\cos 2\pi f_c t - T_s(t)\sin 2\pi f_c t$. Note that $T_c(t)$ and $T_s(t)$ are uncorrelated zero-mean Gaussian random variables at any instant of time.

(b) What is the distribution of ϕ_n?

(c) Are each of the C_q's a random variable or a random process?

(d) Prove that $\overline{|E_z(t)|^2} = (\overline{E_0^2})/2$.

(e) Let $|E_z(t)| = r$. Write the probability distribution of r. What is the name of this distribution?

(f) If the term A cos $2\pi f_c t$ is added to $E_z(t)$, where A is a constant, what type of fading will r undergo?

(g) Using your result in (f), find the K-factor, where $K = A^2/2\sigma^2$, for the case where $A = 5E_0$.

5.18 Derive the RF Doppler spectrum for a 5/8λ vertical monopole receiving a CW signal using the models by Clarke and Gans. Plot the RF Doppler spectrum and the corresponding baseband spectrum out of an envelope detector. Assume isotropic scattering and unit average received power.

5.19 Show that the magnitude (envelope) of the sum of two independent identically distributed complex (quadrature) Gaussian sources is Rayleigh distributed. Assume that the Gaussian sources are zero mean and have unit variance.

5.20 Design a Rayleigh fading simulator: Write a MATLAB program that simulates Rayleigh fading, using the frequency domain method described in Figure 5.24. Assume the maximum Doppler frequency is 200 Hz. Confirm that the level crossing rates and average fade durations of your simulated waveforms agree with Example 5.8. Explain any discrepancies you observed in your simulated outputs. Turn in printouts of your source code, waveform samples, and other pertinent results.

5.21 How would you convert your simulation of Problem 5.20 into a Ricean fading simulator? You do not need to actually do it, just comment on specifics.

5.22 Using the method described in Chapter 5, generate a time sequence of 8192 sample values of a Rayleigh fading signal for

(a) $f_d = 20$ Hz, and
(b) $f_d = 200$ Hz.

5.23 Generate 100 sample functions of fading data described in Problem 5.22, and compare the simulated and theoretical values of the resulting CDF of the data set. Using your data, find R_{RMS}, N_R, and $\bar{\tau}$ for ρ = 1, 0.1, and 0.01. Do your simulations agree with theory? They should!

5.24 Recreate the plots of the CDFs shown in Figure 5.17, starting with the pdfs for Rayleigh, Ricean, and log-normal distributions.

5.25 Plot the probability density function and the CDF for a Ricean distribution having (a) $K = 10$ dB and (b) $K = 3$ dB. The abscissa of the CDF plot should be labeled in dB relative to the median signal level for both plots. Note that the median value for a Ricean distribution changes as K changes.

5.26 Based on your answer in Problem 5.25, if the median RSSI is −70 dBm, what is the likelihood that a signal greater than −80 dBm will be received in a Ricean fading channel having (a) $K = 10$ dB, and (b) $K = 3$ dB?

5.27 The local average power delay profile in a particular environment is found to be

$$P(\tau) = \sum_{n=0}^{2} \frac{10^{-6}}{n^2+1}\delta(\tau - n10^{-6})$$

(a) Sketch the Power Delay Profile of the channel in dBm.
(b) What is the local average power in dBm?
(c) What is the rms delay spread of the channel?
(d) If 256 QAM modulation having a bit rate of 2 Megabits per second is applied to the channel, will the modulation undergo flat or frequency selective fading? Explain your answer.
(e) Over what bandwidth will the channel appear to have constant gain?

5.28 A local spatial average of a power delay profile measured at 900 MHz is shown in Figure P5.28.

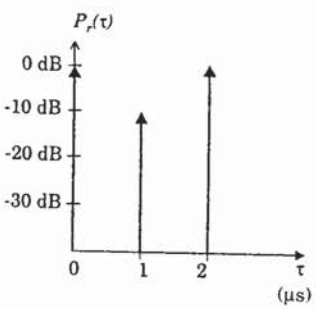

Figure P5.28 Power delay profile.

(a) Determine the rms delay spread and mean excess delay for the channel.

(b) Determine the maximum excess delay (20 dB).

(c) If the channel is to be used with a modulation that requires an equalizer whenever the symbol duration T is less than 10 σ_τ, determine the maximum RF symbol rate that can be supported without requiring an equalizer.

(d) If a mobile traveling at 30 km/hr receives a signal through the channel, determine the time over which the channel appears stationary (or at least highly correlated).

5.29 A flat Rayleigh fading signal at 6 GHz is received by a mobile traveling at 80 km/hr.

(a) Determine the number of positive-going zero crossings about the rms value that occur over a 5 s interval.

(b) Determine the average duration of a fade below the rms level.

(c) Determine the average duration of a fade at a level of 20 dB below the rms value.

5.30 For each of the three scenarios below, decide if the received signal is best described as undergoing fast fading, frequency selective fading, or flat fading.

(a) A binary modulation has a data rate of 500 kbps, f_c = 1 GHz and a typical urban radio channel is used.

(b) A binary modulation has a data rate of 5 kbps, f_c = 1 GHz and a typical urban radio channel is used to provide communications to cars moving on a highway.

(c) A binary modulation has a data rate of 10 bps, f_c = 1 GHz and a typical urban radio channel is used to provide communications to cars moving on a highway.

5.31 Using computer simulation, create a Rayleigh fading simulator that has three independent Rayleigh fading multipath components, each having variable multipath time delay and average power. Then convolve a random binary bit stream through your simulator and observe the time waveforms of the output stream. You may wish to use several samples for each bit (seven is a good number). Observe the effects of multipath spread as you vary the bit period and time delay of the channel.

5.32 Based on concepts taught in this chapter, propose methods that could be used by a base station to determine the vehicular speed of a mobile user. Such methods are useful for handoff algorithms.

5.33 From the shape factor theory described in Section 5.8, describe the physical significance of angular spread, Λ, and determine two examples of $p(\theta)$ that produce: (a) Λ = 1; (b) Λ = 0.

5.34 Using Equation (5.117), determine the coherence distance D_c for each of the four $p(\theta)$ examples you found in Problem 5.33. Note that you must assume a particular direction of travel. Assume λ = 10 cm. How does the direction of travel, relative to the arriving multipath, impact D_c? Explore by proposing four different directions of travel for each of the four $p(\theta)$ functions and determine D_c for each case.