4.14 Problems

4.1 If $P_t = 10$ W, $G_t = 0$ dB, $G_r = 0$ dB, and $f_c = 900$ MHz, find P_r in Watts at a free space distance of 1 km.

4.2 Assume a receiver is located 10 km from a 50 W transmitter. The carrier frequency is 6 GHz and free space propagation is assumed, $G_t = 1$ and $G_r = 1$.

(a) Find the power at the receiver.

(b) Find the magnitude of the E-field at the receiver antenna.

(c) Find the rms voltage applied to the receiver input, assuming that the receiver antenna has a purely real impedance of 50 Ω and is matched to the receiver.

4.3 *Fraunhofer distance*: Calculate the gain, half power beamwidth (HPBW), and Fraunhofer distance for a uniformly illuminated horn antenna at 60 GHz with dimensions of 4.6 cm × 3.5 cm. Hint: HPBW for the horn antenna can be estimated as HPBW $= 51\lambda/a$, where a is the aperture width, as discussed in [Stu81].

4.4 *Free space propagation*: Assume the transmitter power is 1 W at 60 GHz fed into the transmitter antenna. Using the horn antenna from Problem 4.3 at both the transmitter and receiver:

(a) Calculate the free space path loss at 1 m, 100 m, and 1000 m.

(b) Calculate the received signal power at these distances.

(c) What is the rms voltage received at the antenna if the receiver antenna has purely real impedance of 50 Ω and is matched to the receiver?

4.5 *Reflection coefficients*: Find the reflection coefficients for typical ground, brick, limestone, glass, and water from data given in Table 4.1 at an incident angle of 30°. Assume lossless dielectrics. Present your final answers in a table.

4.6 *Surface roughness*: Explain the dependence of surface roughness on the frequency and angle of incidence.

4.7 Show that the Brewster angle (case where $\Gamma_{\parallel} = 0$) is given by θ_i where $\sin\theta_i = \sqrt{\dfrac{\varepsilon_r^2 - \varepsilon_r}{\varepsilon_r^2 - 1}}$.

4.8 (a) Explain the advantages and disadvantages of the two-ray ground reflection model in the analysis of path loss.

(b) In the following cases, tell whether the two-ray model could be applied, and explain why or why not:

$h_t = 35$ m, $h_r = 3$ m, $d = 250$ m
$h_t = 30$ m, $h_r = 1.5$ m, $d = 450$ m

(c) What insight does the two-ray model provide about large-scale path loss that was disregarded when cellular systems used very large cells?

4.9 Prove that in the two-ray ground reflected model, $\Delta = d'' - d' \approx 2h_t h_r / d$. Show when this holds as a good approximation. Hint: Use the geometry of Figure P4.9.

4.10 In a two-ray ground reflected model, assume that θ_Δ must be kept below 6.261 radians for phase cancellation reasons. Assuming a receiver height of 2 m, and given a requirement that θ_i be less than 5°, what are the minimum allowable values for the T–R separation distance and the height of the transmitter antenna? The carrier frequency is 900 MHz. Refer to Figure P4.9.

4.11 In the two-ray path loss model with $\Gamma_{\perp} = -1$, derive an appropriate expression for the location of the signal nulls at the receiver.

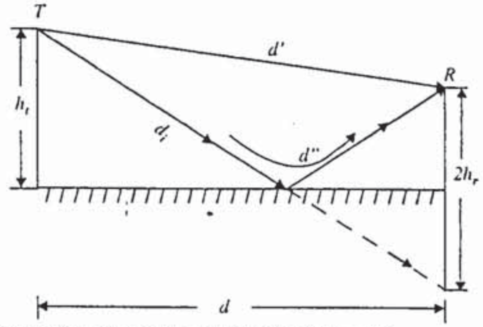

Figure P4.9 Illustration of two-ray ground reflection model.

4.12 Compare the received power for the exact (Equation (4.47)) and approximate (Equation (4.52)) expressions for the two-ray ground reflection model. Assume the height of the transmitter is 40 m and the height of the receiver is 3 m. The frequency is 1800 MHz, and unity gain antennas are used. Plot the received power for both models continuously over the range of 1 km to 20 km, assuming the ground reflection coefficient of −1 for horizontal polarization.

4.13 Redo Problem 4.12 for the case where the ground reflection coefficient is 1 (e.g., vertical polarization).

4.14 Assuming a receiver is located 10 km from a 50 W transmitter. The carrier frequency is 1900 MHz, free space propagation is assumed, $G_t = 1$, $G_r = 2$, find: (a) the power at the receiver; (b) the magnitude of the E-field at the receiver antenna; (c) the open-circuit rms voltage applied to the receiver input assuming that the receiver antenna has a purely real impedance of 50 Ω and is matched to the receiver; (d) find the received power at the mobile using the two-ray ground reflection model assuming the height of the transmitting antenna is 50 m, receiving antenna is 1.5 m above the ground, and the ground reflection is −1.

4.15 Referring to Figure P4.9, compute $d = d_f$, the first Fresnel zone distance between transmitter and receiver for a two-ray ground reflected propagation path, in terms of h_t, h_r, and λ. This is the distance at which path loss begins to transition from d^2 to d^4 behavior. Assume $\Gamma = -1$.

4.16 For the knife-edge geometry in Figure P4.16, show that

(a) $\phi = \dfrac{2\pi\Delta}{\lambda} = \dfrac{2\pi}{\lambda}\left[\dfrac{h^2}{2}\left(\dfrac{d_1 + d_2}{d_1 d_2}\right)\right]$ and

(b) $\upsilon = \alpha\sqrt{\dfrac{2d_1 d_2}{\lambda(d_1 + d_2)}}$ where $\dfrac{\upsilon^2\pi}{2} = \phi$, $d_1, d_2 \gg h$, $h \gg \lambda$, and $\Delta = p_1 + p_2 - (d_1 + d_2)$.

4.17 A general design rule for microwave links is 55% clearance of the first Fresnel zone. For a 1 km link at 2.5 GHz, what is the maximum first Fresnel zone radius? What clearance is required for this system?

4.18 *Diffraction*: From the knife-edge diffraction model, show how the diffracted power depends on frequency. Assume $d_1 = d_2 = 500$ m and $h = 10$ m in Figure P4.16. Hint: You may need to calculate Fresnel's integral or use Lee's approximation, Equations (4.59) and (4.61), respectively.

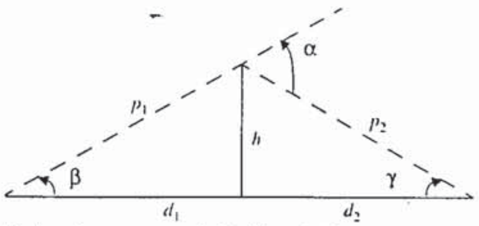

Figure P4.16 Knife-edge geometry for Problem 4.16.

4.19 If $P_t = 10$ W, $G_t = 10$ dB, $G_r = 3$ dB and $L = 1$ dB at 900 MHz, compute the received power for the knife-edge geometry shown in Figure P4.19. Compare this value with the theoretical free space received power if an obstruction did not exist. What is the path loss due to diffraction for this case?

4.20 If the geometry and all other system parameters remain exactly the same in Problem 4.19, but the frequency is changed, redo Problem 4.19 for the case of (a) $f = 50$ MHz and (b) $f = 1900$ MHz.

4.21 *Path loss model*: During the first month of work, you get an assignment to perform a measurement campaign to estimate the channel path loss exponent for a new wireless product. You performed field measurements and collected the following data:

Reference path loss: $PL_0(d_0)$

Path loss measurements: $PL_1(d_1)$, ... $PL_n(d_n)$ at distances: d_1 ... d_n

Using the path loss exponent model from Equation (4.69), find an expression for the optimum value of the path loss exponent n, which minimizes the mean square error between measurements and the model. Hint: The optimum value of n should minimize the mean square error (MSE) between your predicted path loss and measured path loss.

4.22 Assume that local average signal strength field measurements were made inside a building, and post processing revealed that the measured data fit a distant-dependent mean power law model having a log-normal distribution about the mean. Assume the mean power law was found to be $P_r(d) \propto d^{-3.5}$. If a signal of 1 mW was received at $d_0 = 1$ m from the transmitter, and at a distance of 10 m, 10% of the measurements were stronger than −25 dBm, define the standard deviation, σ, for the path loss model at $d = 10$ m.

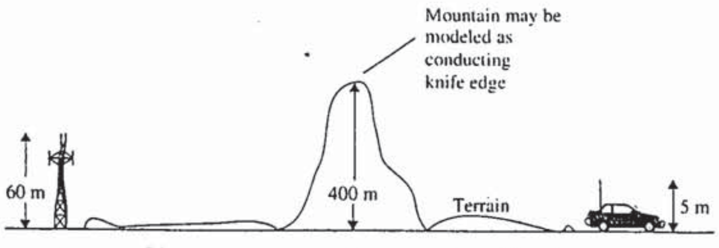

Figure P4.19 Knife-edge geometry for Problem 4.19.

4.23 If the received power at a reference distance $d_0 = 1$ km is equal to 1 microwatt, find the received powers at distances of 2 km, 5 km, 10 km, and 20 km from the same transmitter for the following path loss models: (a) Free space; (b) $n = 3$; (c) $n = 4$; (d) two-ray ground reflection using the exact expression; and (e) extended Hata model for a large city environment. Assume $f = 1800$ MHz, $h_t = 40$ m, $h_r = 3$ m, $G_t = G_r = 0$ dB. Plot each of these models on the same graph over the range of 1 km to 20 km. Comment on the differences between these five models.

4.24 Assume the received power at a reference distance $d_0 = 1$ km is equal to 1 microwatt, and $f = 1800$ MHz, $h_t = 40$ m, $h_r = 3$ m, $G_t = G_r = 0$ dB. Compute, compare, and plot the exact two-ray ground reflection model of (4.47) with the approximate expression given in Equation (4.52). At what T–R separations do the models agree and disagree? What are the ramifications of using the approximate expression instead of the exact expression in cellular system design?

4.25 Suppose that a mobile station is moving along a straight line between base stations BS1 and BS2, as shown in Figure P4.25. The distance between the base stations is $D = 1600$ m. The received power (in dBm) at base station i, from the mobile station, is modeled as (reverse link)

$$P_{r,i}(d) = P_0 - 10n\log_{10}(d_i/d_0) + \chi_i \quad \text{(dBm)} \quad i = 1,2$$

where d_i is the distance between the mobile and base station i, in meters, P_0 is the received power at distance d_0 from the mobile antenna, and n is the path loss exponent. The term $P_0 - 10n\log_{10}(d_i/d_0)$ is usually called *local area mean power*. The terms χ_i are zero-mean Gaussian random variables with standard deviation σ, in dB, that model the variation of the received signals due to shadowing. Assume that the random components χ_i of the signals received at different base stations are independent of each other. n is the path loss exponent.

The minimum usable signal for acceptable voice quality at the base station receiver is $P_{r,min}$, and the threshold level for handoff initiation is $P_{r,HO}$, both in dBm.

Assume that the mobile is currently connected to BS_1. A handoff occurs when the received signal at the base station BS_1, from the mobile, drops below threshold $P_{r,HO}$, and the signal received at candidate base station BS_2 is greater than the minimum acceptable level $P_{r,min}$.

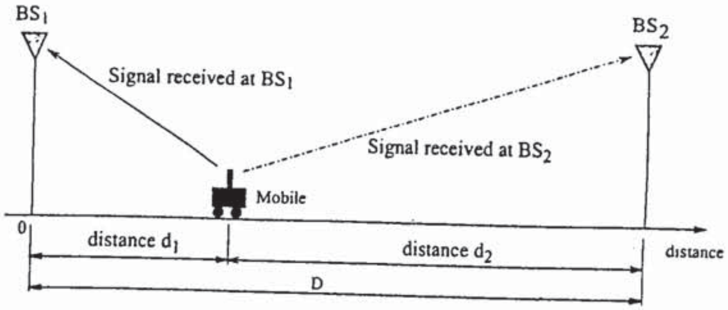

Figure P4.25 Mobile moving along straight line between BS_1 and BS_2.

Table P4.25 Parameters for Problem 4.25

Parameter	Value
n	4
σ	6 dB
P_0	0 dBm
d_0	1 m
$P_{r,min}$	−118 dBm
$P_{r,HO}$	−112 dBm

Using the parameters in Table P4.25, determine:

(a) The probability that a handoff occurs (Pr[handoff]), as a function of the distance between the mobile and its serving base station. Show your result in a plot Pr[handoff] vs. distance d_1.

(b) The distance d_{ho} between base station BS$_1$ and the mobile, such that the probability that a handoff occurs is equal to 80%.

4.26 Derive Equations (4.78) and (4.79) from first principles and reproduce some of the curves on Figure 4.18.

4.27 A transmitter provides 15 W to an antenna having 12 dB gain. The receiver antenna has a gain of 3 dB and the receiver bandwidth is 30 kHz. If the receiver system noise figure is 8 dB and the carrier frequency is 1800 MHz, find the maximum T–R separation that will ensure that a SNR of 20 dB is provided for 95% of the time. Assume $n = 4$, $\sigma = 8$ dB, and $d_0 = 1$ km.

4.28 Assume a SNR of 25 dB is desired at the receiver. If a 900 MHz cellular transmitter has an EIRP of 100 W, and the AMPS receiver uses a 0 dB gain antenna and has a 10 dB noise figure, find the percentage of time that the desired SNR is achieved at a distance of 10 km from the transmitter. Assume $n = 4$, $\sigma = 8$ dB, and $d_0 = 1$ km.

4.29 Four received power measurements were taken at distances of 100 m, 200 m, 1 km, and 2 km from a transmitter. The measured values at these distances are −0 dBm, −25 dBm, −35 dBm, and −38 dBm, respectively. It is assumed that the path loss for these measurements follows the model

$$PL(d)[\mathrm{dB}] = \overline{PL}(d) + X_\sigma = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma$$

where $d_0 = 100$ m.

(a) Find the minimum mean square error (MMSE) estimate for the path loss exponent, n.

(b) Calculate the standard deviation of shadowing about the mean value.

(c) Estimate the received power at $d = 2$ km using the resulting model.

(d) Predict the likelihood that the received signal level at 2 km will be greater than −35 dBm. Express your answer as a Q-function.

4.30 Read "Path Loss, Delay Spread, and Outage Models as Functions of Antenna Height for Microcellular System Design," which appeared in the August 1994 issue of *IEEE Transactions on Vehicular Technology*, Vol. 43, No. 3. (This paper also appears on page 211 of *Cellular Radio and Personal Communications: Advanced Selected Readings* by the IEEE Press.)

(a) Derive Equation (10) in the paper from first principles and from the definition of Fresnel zone.

(b) Using the models for mean path loss given in the paper, determine the received signal powers at T–R separation distances of 50 m, 100 m, and 1 km for a 1 W transmitter and unity gain antennas operating at 1900 MHz for:

(1) obstructed environments

(2) LOS environments

Assume an 8.5 m tall base station antenna and a 1.7 m tall mobile antenna are used.

(c) For your answers in (b), use a log-normal shadowing model to determine the likelihood that the received signal will be greater than or equal to −70 dBm at each of the three T–R separations, for obstructed and for LOS environments.

(d) Using the overbound model for rms delay spread given on pages 493 of the journal paper, estimate the rms delay spread at each of the three T–R separation distances, for obstructed and for LOS environments.

(e) Using your values in (d), determine the maximum unequalized symbol data rates that may be successfully transmitted by the base station at each of the three T–R separation distances in obstructed and in LOS environments. Assume an adaptive equalizer is not needed at the mobile receiver when the symbol duration is greater than 10 times the rms delay spread, and assume the noise figures of IS-136 and GSM mobile receivers are 6 dB. Would it be possible to receive IS-136 signals at these distances and environments without an equalizer? Would it be possible to transmit GSM signals at these distances without an equalizer? Hint: Consider both time delay spread and thermal noise-limited receiver power to provide answers.

4.31 Design and create a computer program that produces an arbitrary number of samples of propagation path loss using a d^n path loss model with log normal shadowing. Your program is a radio propagation simulator, and should use, as inputs, the T–R separation, frequency, the path loss exponent, the standard deviation of the log-normal shadowing, the close-in-reference distance, and the number of desired predicted samples. Your program should provide a check that insures that the input T–R separation is equal to or exceeds the specified input close-in-reference distance, and should provide a graphical output of the produced samples as a function of path loss and distance (this is called a *scatter plot*).

Verify the accuracy of your computer program by running it for 50 samples at each of five different T–R separation distances (a total of 250 predicted path loss values), and determine the best fit path loss exponent and the standard deviation about the mean path loss exponent of the predicted data using the techniques described in Example 4.9. Draw the best fit mean path loss model on the scatter plot to illustrate the fit of the model to the predicted values. You will know your simulator is working if the best fit path loss model and the standard deviation for your simulated data is equal to the parameters you specified as inputs to your simulator.

4.32 Using the computer program developed in Problem 4.31, develop an interface that allows a user to specify inputs as described in Problem 4.31, as well as transmitter and receiver parameters such as transmit power, transmit antenna gain, receiver antenna gain, receiver bandwidth, and receiver noise figure. Using these additional input parameters, and using knowledge of the

Q-function and noise calculations (see Appendices), you may now statistically determine coverage levels for any specific mobile radio system. You may wish to implement table look-ups for *Q* and *erf* functions so that your simulator provides answers to the following wireless system design problems:

(a) If a user specifies all input parameters listed above, and specifies a desired received SNR and a specific value of T–R separation distance, what is the percentage of time that the SNR will be exceeded at the receiver?

(b) If a user specifies all input parameters listed above, and specifies a desired percentage of time that the SNR will be exceeded at the receiver, then what is the maximum value of T–R separation that will meet or exceed the specified percentage?

(c) If a user specifies a particular percentage that a given SNR is provided for a particular T–R separation d (assumed to be on the boundary of a cell), then what is the percentage of area that will be covered within the cell having the same radius d?

(d) Handle questions (a)-(c) above, except for the case where the user wishes to specify the received signal power level (in dBm) instead of specifying SNR.

Verify the functionality of your simulator by example.

4.33 A PCS licensee plans to build out a 30 MHz license in the new U.S. PCS band of 1850 MHz to 1880 MHz (reverse link) and 1930 MHz to 1960 MHz (forward link). They intend to use DCS 1900 radio equipment. DCS 1900 provides a GSM-like service and supports eight users per 200 kHz radio channel using TDMA. Because of GSM's digital techniques, GSM vendors have convinced the licensee that when the path loss exponent is equal to four, GSM can be deployed using four-cell reuse.

(a) How many GSM radio channels can be used by the licensee?

(b) If each DCS 1900 base station can support a maximum of 64 radio channels, how many users can be served by a base station during fully loaded operation?

(c) If the licensee wishes to cover a city having a circular shaped area of 2500 sq km, and the base stations use 20 W transmitter powers and 10 dB gain omnidirectional antennas, determine the number of cells required to provide forward link coverage to all parts of the city. Assume four-cell reuse, and let $n = 4$ and the standard deviation of 8 dB hold as the path loss model for each cell in the city. Also assume that a required signal level of –90 dBm must be provided for 90% of the coverage area in each cell, and that each mobile uses a 3 dBi gain antenna. Assume $d_o = 1$ km.

(d) For your answer in (c), define in exact detail a suitable channel reuse scheme for each cell in the city, and define the channels used by each cell. Your scheme should include details such as how many channels each base station should use, what the nearest reuse distance should be, and other issues which clearly define how to assign channels geographically throughout the city? You may assume that users are distributed uniformly throughout the city, that each cell is equidistant from its neighbors, and you may ignore the effect of control channels (that is, assume all radio channels carry only voice users).

(e) How many (i) cells (base stations), (ii) total radio channels, and (iii) total user channels (there are eight user channels per radio channel) are available throughout the entire city, based on your answer in (d)? The total number of user channels is equal to the maximum capacity of the system and is a hard limit on the number of users that can be simultaneously served at full capacity.

(f) If each base station costs \$500,000, and each radio channel within the base station costs \$50,000, what is the cost of the system in (e)? This is the initial cost of the system.

(g) If the system in (d) is designed for 5% blocking probability at start-up, what is the maximum number of subscribers that can be supported at start-up? This is the number of phones that may be initially subscribed at start-up. Assume that each user channel is trunked along with the other user channels on other radio channels within the base station.

(h) Using your answer in (g), what is the average cost per user needed to recoup 10% of the initial system buildout cost after one year if the number of subscribers is static during year 1?

4.34 Consider seven-cell frequency reuse. Cell B1 is the desired cell and B2 is a co-channel cell as shown in Figure P4.34(a). For a mobile located in cell B1, find the minimum cell radius R to give a forward link C/I ratio of at least 18 dB at least 99% of the time. Assume the following:

Co-channel interference is due to base B2 only.

Carrier frequency, $f_c = 890$ MHz.

Reference distance, $d_0 = 1$ km (assume free space propagation from the transmitter to d_0).

Assume omnidirectional antennas for both transmitter and receiver, where $G_{base} = 6$ dBi and $G_{mobile} = 3$ dBi.

Transmitter power, $P_t = 10$ W (assume equal power for all base stations).

PL(dB) between the mobile and base B1 is given as

$$\overline{PL}(\mathrm{dB}) = \overline{PL}(d_0) + 10(2.5)\log\left(\frac{d_1}{d_0}\right) - X_\sigma \qquad \sigma = 0\ \mathrm{dB}.$$

PL(dB) between the mobile and base B2 is given as

$$\overline{PL}(\mathrm{dB}) = \overline{PL}(d_0) + 10(4.0)\log\left(\frac{d_2}{d_0}\right) - X_\sigma \qquad \sigma = 7\ \mathrm{dB}.$$

Cell boundaries are shown in the Figure P4.34(b).

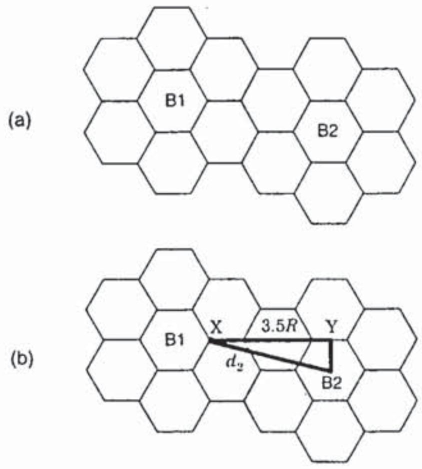

Figure P4.34 (a) Seven-cell reuse structure; (b) co-channel interference geometry between B1 and B2.